



# Fraction Tower® comparing and ordering fractions



## Objectives

Students will...

- Prove that certain fractions are  $>$ ,  $<$ , or  $=$  to other given fractions
- Order fractions from greatest to least and least to greatest
- Name several fractions that are equivalent to a given fraction

## Materials

- Fraction Tower® Cubes (TB17368)
- Worksheets (attached with lesson plan download)

## Learning standards

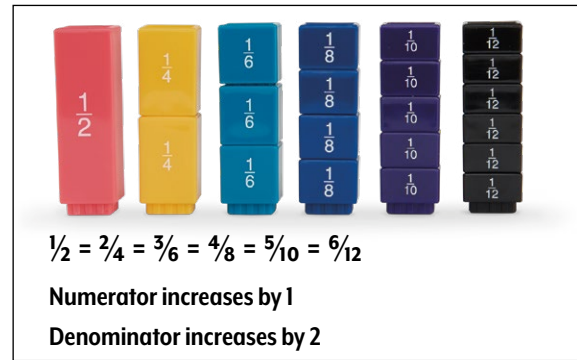
- Determine if two given fractions are equivalent using a variety of methods.
- Compare two fractions with different numerators and different denominators and represent the comparison using the symbols  $>$ ,  $<$ , or  $=$ .

## Introduction

1. Distribute Worksheet 1. Have students pick up their red tower and ask how many pieces are in the red tower (1). This represents one whole.
2. Students should now look at the pink tower and count how many pieces it has (2). Ask students how many pink pieces are needed to make the same height as the red tower (2). Students should be able to see that, when standing side by side, they are exactly the same height. This means that  $\frac{2}{2}$  is equivalent to 1. Have them write  $\frac{2}{2}$  on their worksheet for problem 1.
3. Have students count the number of pieces in the yellow tower (4). Ask how many yellow pieces are needed to make the same height as the red tower (4). Again, when standing side by side, the towers are exactly the same height, meaning that  $\frac{4}{4}$  is equivalent to 1. Students should write  $\frac{4}{4}$  on the worksheet for problem 1 as another fraction that is equivalent to 1.
4. Follow the same procedure with the teal tower and the blue tower. The teal tower has 6 pieces, meaning that  $\frac{6}{6}$  is equivalent to 1. The blue tower has eight pieces, making  $\frac{8}{8}$  equivalent to 1 as well. Students should also record  $\frac{6}{6}$  and  $\frac{8}{8}$  on their worksheet for problem 1. Students should begin to detect a pattern.
5. Continue with the purple tower (10 pieces) and the black tower (12 pieces). Be sure that students add  $\frac{10}{10}$  and  $\frac{12}{12}$  to problem 1 on the worksheet. Students should be catching on to the pattern and realizing that when a fraction has the same number in the numerator and in the denominator, then the fraction is equivalent to 1 whole. Have students come up with three more fractions that are equivalent to 1 and list them on the worksheet for problem 1.

# Activity 1

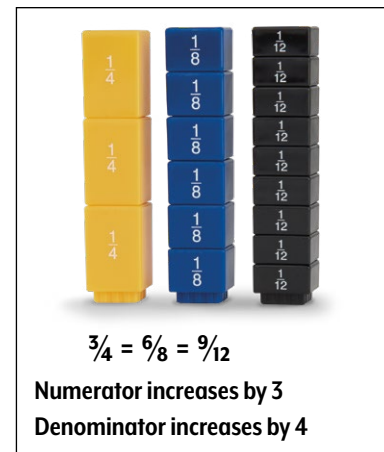
1. Start with one of the pink tower pieces. Ask what fraction that piece represents ( $\frac{1}{2}$ ).
2. Have students break apart the pieces of the yellow tower, then ask if they can make a tower that's exactly the same height as the pink half tower using just yellow pieces (yes) and how many yellow pieces it will take (2). If it takes two pieces of the tower that's been split into fourths, students should be able to determine that they just made the fraction  $\frac{2}{4}$ . Inform students that the numerator of the fraction represents the number of pieces that they used, and the denominator is the number of total pieces in the whole tower. In this case, 2 is the numerator and 4 is the denominator. Have the students write  $\frac{2}{4}$  as an equivalent fraction to  $\frac{1}{2}$  on their worksheet for problem 2.
3. Have students break apart the teal tower. Ask if they can make a tower that's exactly the same height as the pink and yellow towers that they just made. They should say they can and it takes three teal pieces to do so. Now point out that they've used three pieces of the teal tower and that it is split into sixths. Ask what fraction they've made ( $\frac{3}{6}$ ). Add  $\frac{3}{6}$  to problem 2 of the worksheet.
4. Guide students through the same procedure with the blue tower. They should be able to use four blue pieces to make a tower that's the same height as the pink, yellow, and teal towers they've already made. Point out that they've used four pieces of the blue tower, which has been split into eighths. They should be able to say they made the fraction  $\frac{4}{8}$ . Add  $\frac{4}{8}$  to problem 2 of the worksheet.



Steps 1-5

5. Have students continue doing the same thing with the purple and black towers on their own. After they have done this, see if they have discovered that  $\frac{5}{10}$  and  $\frac{6}{12}$  are also equivalent to  $\frac{1}{2}$ . Ask them what is happening to the numerator each time, then ask about the denominator. (*The numerator goes up by 1 each time, and the denominator goes up by 2 each time.*) Following this same pattern, they should be able to say that the next fraction in the group would be  $\frac{7}{14}$ . Students should then write the next three terms in the pattern on their own ( $\frac{8}{16}$ ,  $\frac{9}{18}$ , and  $\frac{10}{20}$ ). Make sure they understand that all of these fractions are equivalent to  $\frac{1}{2}$ . Students should now be able to complete the rest of problem 2.

6. Now students should use three yellow pieces to make a tower. Ask what fraction this represents and if they can make an equivalent fraction using the pink tower ( $\frac{3}{4}$ , no). Ask why they can't do this. (*One pink piece is too small and two pink pieces is too big.*) Explain that this means no fraction with a denominator of 2 is equivalent. With  $\frac{1}{2}$ , it was possible to make an equivalent fraction with every Fraction Tower®, but that's not the way it always works.
7. Ask if an equivalent fraction can be made using the teal tower. Have students break the pieces apart and try to build a tower that's the same size as the yellow  $\frac{3}{4}$  tower. They will discover that it doesn't work. Based on their experimentation, ask which fraction is just a little bit less than  $\frac{3}{4}$  ( $\frac{4}{6}$ ), then which fraction is a little bit greater than  $\frac{3}{4}$  ( $\frac{5}{6}$ ).
8. Have students break the pieces apart on the blue tower and see if they can build a tower the same size as the yellow  $\frac{3}{4}$  tower. They should be able to use six blue pieces and realize that if they used six pieces and the full blue tower is eight pieces, the fraction is  $\frac{6}{8}$ . Students should record that fraction on problem 3 of the worksheet.
9. Next, use the pieces of the purple tower to try to make an equivalent fraction to  $\frac{3}{4}$ . When they are unable to, ask which fraction is a little bit less than  $\frac{3}{4}$  ( $\frac{7}{10}$ ) and which fraction is a little bit greater than  $\frac{3}{4}$  ( $\frac{8}{10}$ ).
10. Move on to the black tower and see if those pieces can make a fraction equivalent to  $\frac{3}{4}$ . Students should be able to use nine black pieces and see that if they've used nine pieces and the full black tower has 12 pieces, the fraction is  $\frac{9}{12}$ . That fraction should also be recorded on problem 3 of the worksheet.

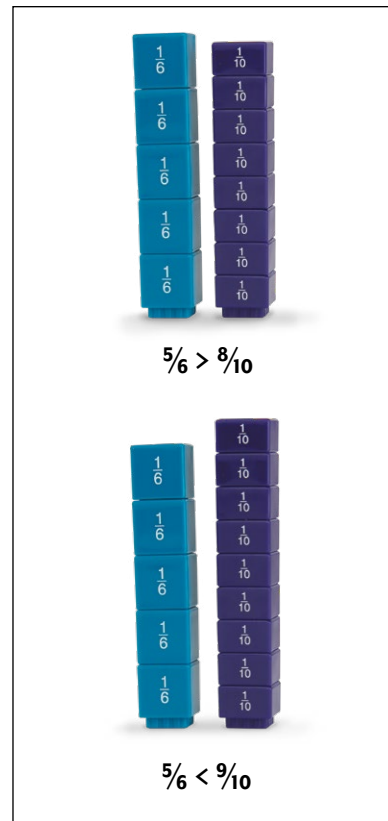


Steps 6-11

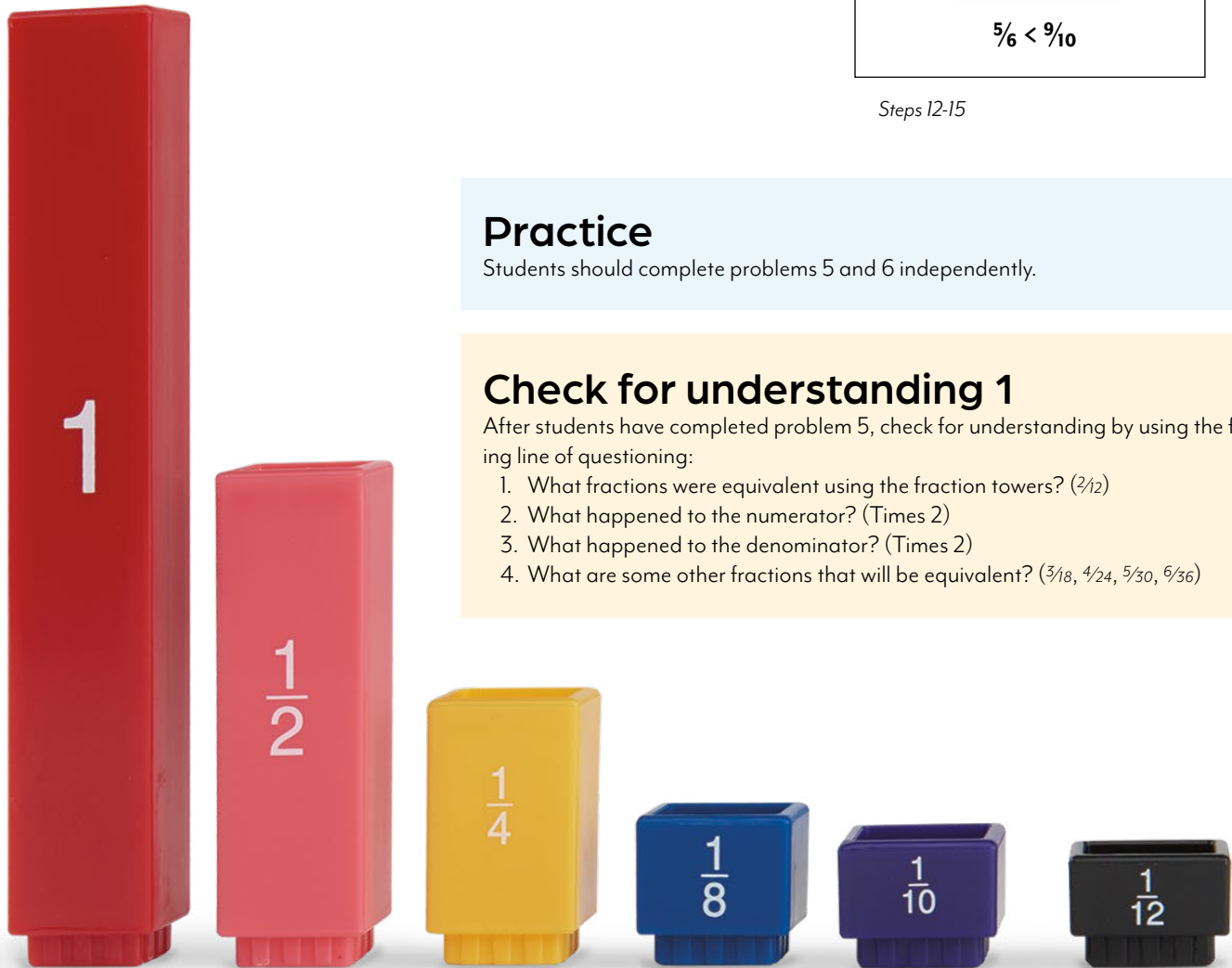
11. Get students to realize the emerging pattern by asking what's happening to the numerator and the denominator each time. (*The numerator goes up by 3 and the denominator goes up by 4 each time.*) See if students can identify the next equivalent fraction ( $\frac{12}{16}$ ). Have students complete the rest of problem 3 on their worksheet, including writing the next three fractions that are equivalent to  $\frac{3}{4}$  ( $\frac{15}{20}$ ,  $\frac{18}{24}$ , and  $\frac{21}{28}$ ).

## Activity 1 (continued)

12. Begin the last equivalent fraction example by having students use five teal pieces to make a tower. If students use five of the six teal pieces to make a tower, they should be able to say that they are using  $\frac{5}{6}$  of the teal pieces.
13. See if students can make an equivalent fraction to  $\frac{5}{6}$  using the pink pieces. When they can't, have them explain why not ( $\frac{1}{2}$  is less than  $\frac{5}{6}$  and  $\frac{2}{2}$  is greater than  $\frac{5}{6}$ ). Continue the same procedure with the yellow, blue, and purple pieces. Each time, they should explain why they can't use those pieces to form a fraction equivalent to  $\frac{5}{6}$ .
- **Yellow:**  $\frac{3}{4}$  is less than  $\frac{5}{6}$  and  $\frac{4}{4}$  is greater than  $\frac{5}{6}$ .
  - **Blue:**  $\frac{6}{8}$  is less than  $\frac{5}{6}$  and  $\frac{7}{8}$  is greater than  $\frac{5}{6}$ .
  - **Purple:**  $\frac{8}{10}$  is less than  $\frac{5}{6}$  and  $\frac{9}{10}$  is greater than  $\frac{5}{6}$ .
14. Follow the same procedure with the black pieces. Students will find that they can use 10 black pieces to make an equivalent tower, meaning that the equivalent fraction would be  $\frac{10}{12}$ . They should add this fraction to the worksheet for problem 4.
15. Point out that even though they only have two fractions, they can still determine a pattern. First, ask what happened to the numerator from the first fraction to the second. Remind them that they've been multiplying all day (*times 2*). Then ask what happened to the denominator from the first fraction to the second (*times 2*). Ask what the next two fractions in the pattern would be if the same rule was kept ( $\frac{20}{24}$  and  $\frac{40}{48}$ ). They should now be able to complete the rest of problem 4.



Steps 12-15



## Practice

Students should complete problems 5 and 6 independently.

## Check for understanding 1

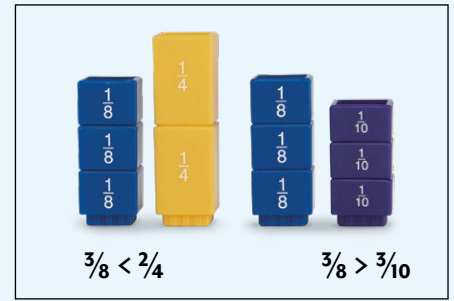
After students have completed problem 5, check for understanding by using the following line of questioning:

1. What fractions were equivalent using the fraction towers? ( $\frac{2}{12}$ )
2. What happened to the numerator? (Times 2)
3. What happened to the denominator? (Times 2)
4. What are some other fractions that will be equivalent? ( $\frac{3}{18}$ ,  $\frac{4}{24}$ ,  $\frac{5}{30}$ ,  $\frac{6}{36}$ )

## Activity 2

This activity builds on students' understanding of equivalent fractions by adding  $>$  and  $<$  to the mix. Hand out Worksheet 2 prior to beginning the activity.

1. Begin by creating a tower with three blue pieces. Ask what fraction this tower represents ( $\frac{3}{8}$ ).
2. Students will now use pieces from their yellow tower to see if they can make any fractions that are less than  $\frac{3}{8}$ . Tell them that they'll know it's less than  $\frac{3}{8}$  because the tower will be shorter than the blue  $\frac{3}{8}$  tower. Students should be able to make a shorter tower and say that the fraction that their yellow tower represents is  $\frac{1}{4}$ . Using the tower heights, they have been able to prove that  $\frac{1}{4} < \frac{3}{8}$ . Have them write the fraction in the appropriate place for problem 1 of the worksheet.
3. Now students will see if they can make a tower with their yellow pieces that's greater than  $\frac{3}{8}$ . Remind them that they will know it's greater because the yellow tower will be taller than the  $\frac{3}{8}$  blue tower. They should be able to make three different towers that are greater than  $\frac{3}{8}$  ( $\frac{2}{4}$ ,  $\frac{3}{4}$ , and  $\frac{4}{4}$ ). These heights all prove that  $\frac{2}{4} > \frac{3}{8}$ ,  $\frac{3}{4} > \frac{3}{8}$ , and  $\frac{4}{4} > \frac{3}{8}$ . Students should select one of these equations to record on their worksheet for problem 1.
4. For the next step, students will continue to work with the blue  $\frac{3}{8}$  tower, but now they will use the purple pieces and work with fractions that use tenths. Have students use their purple pieces to see if they can make a fraction that is less than  $\frac{3}{8}$ . Remind them that they will know a fraction is less than  $\frac{3}{8}$  because the purple tower will be shorter than the blue  $\frac{3}{8}$  tower. They should be able to make three different fractions that are less than  $\frac{3}{8}$  ( $\frac{1}{10}$ ,  $\frac{2}{10}$ , and  $\frac{3}{10}$ ). Students should select one of these fractions to record on their worksheet for problem 1.
5. Students will now try to make a purple tower that shows a fraction greater than  $\frac{3}{8}$ . Again, remind them that they'll know it's greater than  $\frac{3}{8}$  because the purple tower will be taller than the blue  $\frac{3}{8}$  tower. There are multiple towers students should be able to make that represent fractions that are greater than  $\frac{3}{8}$  ( $\frac{4}{10}$ ,  $\frac{5}{10}$ ,  $\frac{6}{10}$ ,  $\frac{7}{10}$ ,  $\frac{8}{10}$ ,  $\frac{9}{10}$ , and  $\frac{10}{10}$ ). Students may record any one of those fractions on their worksheet for problem 1.
6. Keeping the blue  $\frac{3}{8}$  tower, students will now follow the same procedure with their teal pieces to see if they can make any fractions that are less than  $\frac{3}{8}$ . They should be able to make one of two towers ( $\frac{1}{6}$  or  $\frac{2}{6}$ ). Reinforce the idea that students can tell these fractions are less than  $\frac{3}{8}$  because the tower is shorter than the blue  $\frac{3}{8}$  tower. Students should record one of the fractions on their worksheet for problem 1. For fractions greater than  $\frac{3}{8}$ , the teal tower will be taller, and students should be able to make one of four different towers ( $\frac{3}{6}$ ,  $\frac{4}{6}$ ,  $\frac{5}{6}$ , or  $\frac{6}{6}$ ). Students should also record one of these fractions on their worksheet for problem 1.



## Check for understanding 2

Have students complete problem 2 on their worksheet on their own. When all are done, see if they can answer the following questions:

1. What are some fractions that are greater than  $\frac{7}{10}$ ? ( $\frac{3}{4}$ ,  $\frac{5}{6}$ ,  $\frac{6}{8}$ ,  $\frac{7}{8}$ ,  $\frac{9}{12}$ ,  $\frac{10}{12}$ , and  $\frac{11}{12}$ )
2. What are some fractions that are less than  $\frac{7}{10}$ ? ( $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{2}{4}$ ,  $\frac{1}{6}$ ,  $\frac{2}{6}$ ,  $\frac{3}{8}$ ,  $\frac{3}{12}$ , etc.)
3. How do you know when a fraction is greater than a given fraction? (*The tower is taller.*)
4. How do you know when a fraction is less than a given fraction? (*The tower is shorter.*)

Students should complete the rest of the worksheet independently.

## Intervention

1. Only use the 1,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{6}$ , and  $\frac{1}{8}$  towers.
2. Concentrate on  $>$ ,  $<$ , or  $=$ , rather than also including ordering fractions
3. Stick to Activity 1 and provide more opportunities for practice with other fractions.

## Extension

1. Have students create their own  $>$ ,  $<$ , or  $=$  problems.
2. Have students order four fractions from least to greatest.
3. Have students begin exploration of adding fractions.

Name: \_\_\_\_\_ Date: \_\_\_\_\_

1. What are some fractions that are equivalent to 1? \_\_\_\_\_

Come up with three more fractions that are equivalent to 1. \_\_\_\_\_

2. Using the Fraction Towers®, what fractions are equivalent to  $\frac{1}{2}$ ? \_\_\_\_\_

What happens to the numerator? \_\_\_\_\_

What happens to the denominator? \_\_\_\_\_

What other fractions are equivalent to  $\frac{1}{2}$ ? \_\_\_\_\_

3. Using the Fraction Towers®, what fractions are equivalent to  $\frac{3}{4}$ ? \_\_\_\_\_

What happens to the numerator? \_\_\_\_\_

What happens to the denominator? \_\_\_\_\_

What other fractions are equivalent to  $\frac{3}{4}$ ? \_\_\_\_\_

4. Using the Fraction Towers®, what fractions are equivalent to  $\frac{5}{6}$ ? \_\_\_\_\_

What happens to the numerator? \_\_\_\_\_

What happens to the denominator? \_\_\_\_\_

What other fractions are equivalent to  $\frac{5}{6}$ ? \_\_\_\_\_

5. Using the Fraction Towers®, what fractions are equivalent to  $\frac{1}{6}$ ? \_\_\_\_\_

What happens to the numerator? \_\_\_\_\_

What happens to the denominator? \_\_\_\_\_

What other fractions are equivalent to  $\frac{1}{6}$ ? \_\_\_\_\_

6. Using the Fraction Towers®, what fractions are equivalent to  $\frac{1}{4}$ ? \_\_\_\_\_

What happens to the numerator? \_\_\_\_\_

What happens to the denominator? \_\_\_\_\_

What other fractions are equivalent to  $\frac{1}{4}$ ? \_\_\_\_\_

# Fraction Tower® — worksheet 2

Volume 6

Name: \_\_\_\_\_ Date: \_\_\_\_\_

**Directions:** Use three different Fraction Towers® to complete the following greater than and less than statements.

1.  $\frac{3}{8} > \underline{\hspace{2cm}}$

$\frac{3}{8} < \underline{\hspace{2cm}}$

2.  $\frac{7}{10} > \underline{\hspace{2cm}}$

$\frac{7}{10} < \underline{\hspace{2cm}}$

$\frac{3}{8} > \underline{\hspace{2cm}}$

$\frac{3}{8} < \underline{\hspace{2cm}}$

$\frac{7}{10} > \underline{\hspace{2cm}}$

$\frac{7}{10} < \underline{\hspace{2cm}}$

$\frac{3}{8} > \underline{\hspace{2cm}}$

$\frac{3}{8} < \underline{\hspace{2cm}}$

$\frac{7}{10} > \underline{\hspace{2cm}}$

$\frac{7}{10} < \underline{\hspace{2cm}}$

3.  $\frac{1}{2} > \underline{\hspace{2cm}}$

$\frac{1}{2} < \underline{\hspace{2cm}}$

4.  $\frac{5}{12} > \underline{\hspace{2cm}}$

$\frac{5}{12} < \underline{\hspace{2cm}}$

$\frac{1}{2} > \underline{\hspace{2cm}}$

$\frac{1}{2} < \underline{\hspace{2cm}}$

$\frac{5}{12} > \underline{\hspace{2cm}}$

$\frac{5}{12} < \underline{\hspace{2cm}}$

$\frac{1}{2} > \underline{\hspace{2cm}}$

$\frac{1}{2} < \underline{\hspace{2cm}}$

$\frac{5}{12} > \underline{\hspace{2cm}}$

$\frac{5}{12} < \underline{\hspace{2cm}}$

5.  $\frac{5}{6} > \underline{\hspace{2cm}}$

$\frac{5}{6} < \underline{\hspace{2cm}}$

$\frac{5}{6} > \underline{\hspace{2cm}}$

$\frac{5}{6} < \underline{\hspace{2cm}}$

$\frac{5}{6} > \underline{\hspace{2cm}}$

$\frac{5}{6} < \underline{\hspace{2cm}}$