Nasco Education

LESSON PLAN

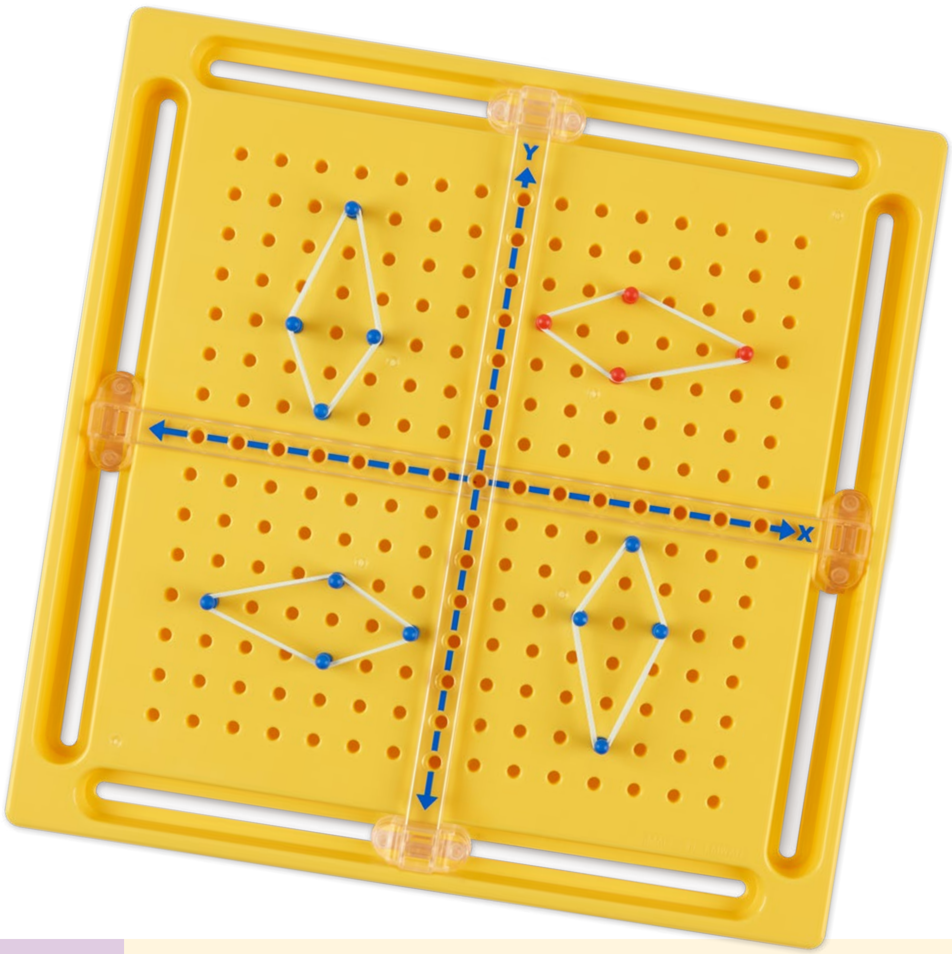
Math

Developed with Kristin Hotter

Rotations in the coordinate plane

Volume 26 | Gr. 6-8

Time: 45-60 mins.



Content

Use the X-Y Coordinate Geoboard to examine and investigate rotations of 90°, 180°, and 270°. Students will be asked to create a particular figure in a given quadrant and then rotate it around the origin (0, 0) into various other quadrants. Students will begin to see the correlation of coordinates as they work on a variety of rotations. Figures for this activity include triangles and quadrilaterals.

Before introducing this lesson to students, it is important that students have a firm grasp of translation and reflections. It is recommended that you complete Nasco MathWorks Volume 25: Translations and Reflections lesson plan (go to nascoeducation.com to download) prior to tackling this one. In that lesson, not only did students gain a firm understanding of translations and reflections, but they also familiarized themselves with coordinate plane terms such as quadrants and axes.

Objectives

Students will...

- Analyze how *x-y* coordinates change through various steps in rotations
- Compare different degrees of rotations
- Conclude that each type of rotation follows a given set of rules

Materials

- X-Y Coordinate Geoboard (**TB24598**) or X-Y Coordinate Geoboard Set of 30 (**TB25306**)
  NOTE: Each board includes pegs and rubber bands.
- Student worksheet and answer key (attached with lesson plan download)

Common core state standards

CCSS.Math.Content.6.G.A.3 — Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

CCSS.Math.Content.8.G.A.1 — Verify experimentally the properties of rotations, reflections, and translations.

Introduction

1. Review translations and reflections. Remind students that translations occur when they move or slide all of the coordinates in a given figure the exact same distance and direction across a given line, such as the *x*-axis or the *y*-axis. Use an X-Y Coordinate Geoboard or an interactive whiteboard to show the following quick example to students.
2. Create a triangle with the coordinates (4, 4), (2, 1), and (6, 1). Explain to students what you are doing to students as you do it. Mention this triangle is in Quadrant I of the coordinate plane. Tell them that you would like to move this figure into Quadrant IV. Ask the following questions as you go through the translation:

   What axis will I need to cross in order to make the translation? ***(the x-axis)***
   How many rows above the *x*-axis are the lowest points of this triangle? ***(1 row)***
   How many rows down will I need to translate the top of my triangle so it is exactly one row from the *x*-axis in Quadrant IV? ***(5 rows)***
   What will the new coordinates be? ***(4, –1)***
   How far down will I need to move the peg that is at (2, 1)? ***(5 rows)***
   What will its new coordinates be? ***(2, –4)***
   How far down will I need to move the peg that is at (6, 1)? ***(5 rows)***
   What will its new coordinates be? (***6, –4)***
   What do you notice about each of the new coordinate pairs? ***(The x-coordinate has remained the same, while the y-coordinate has been changed.)***

   Point out what happens in translations. The coordinate for the axis that is being crossed, in this case the *x*-axis, is the coordinate that remains the same while the opposite coordinate, in this case the *y*-coordinate, has changed.
3. Move on to demonstrating a reflection. Create a triangle on the geoboard or interactive whiteboard with the coordinates (–2, 2), (–6, 2), and (–2, 6). Tell students that your goal is to reflect that triangle into Quadrant I. Remind students that reflect means that they create a flipped mirror image of the original figure. Ask the following questions before beginning the reflection:

   What axis will I need to cross in order to make this reflection? ***(the y-axis)***
   How do we find the reflected coordinate of a given point? ***(find the opposite of one of the coordinates)***
   How do we know which coordinate we will find the opposite of? ***(When we cross the y-axis, we find the opposite of the x-coordinate. When we cross the x-axis, we find the opposite of the y-coordinate.)***

   Tell students that since you are crossing the *y*-axis, you need to find the opposite of the *x*-coordinates. The *y*-coordinates will stay exactly the same.
4. Start with (–2, 2) and ask what the reflected coordinate will be. [It will be ***(2, 2)*** because the opposite of –2 is 2 for the *x*-coordinate, and the *y*-coordinate stays the same at 2.]
5. Move on to (–6, 2). The reflected coordinate will be (6, 2) because the opposite of –6 is 6 for the *x*-coordinate, and the *y*-coordinate stays the same at 2.
6. Finish with (–2, 6). The reflected coordinate will be (2, 6) because the opposite of –2 is 2 for the *x*-coordinate, and the *y*-coordinate stays the same at 6.
7. Tell students that translations and reflections each have a rule that the coordinate opposite of the axis being crossed is the one that changes for those transformations. They should keep those rules in mind as they begin to investigate rotations. They should be thinking about if the same rule applies for rotations or if a new rule will come into play.

Activity

1. Distribute the worksheet and geoboards. Have students build a red triangle in Quadrant I by putting a peg at the following coordinates: (3, 2), (6, 2), and (3, 5). They should also put a small rubber band around their figure. On their worksheet, they should write the coordinates on the line labeled Quadrant I for Figure 1, then draw the triangle they have created in Quadrant I of the coordinate plane for Figure 1.
2. Before they begin investigating and experimenting with rotations, tell students that there are a few rules that will help them along the way. Some of these rules will be discussed now, others as they investigate, and the rest after they investigate. The first rule is that they will always rotate around the origin across either the *x*-axis or the *y*-axis. Each rotation will always be made in a counterclockwise direction. NOTE: The next step provides an overview of what students will be doing. They shouldn't actually be doing any rotation at this time. You are merely guiding them through the process.
3. The original figure is in Quadrant I. Ask which axis will be crossed if you start there and begin to move in a counterclockwise direction ***(y-axis)***. If we cross the *y*-axis, which quadrant will we have rotated into ***(Quadrant II)***? From that quadrant, ask which axis you cross ***(x-axis)***, and if you continue to move in a counterclockwise direction which quadrant you will rotate into ***(Quadrant III)***? Move in a counterclockwise direction from Quadrant III and ask which axis is crossed now ***(y-axis)*** and which quadrant has been rotated into ***(Quadrant IV)***. To finish the rotation from Quadrant IV, students should be able to tell you that you would cross the *x*-axis and rotate into Quadrant I. Once the figure completes the rotation and is back in Quadrant I, it is back at its original location on the coordinate plane, indicating it has completed a complete rotation.

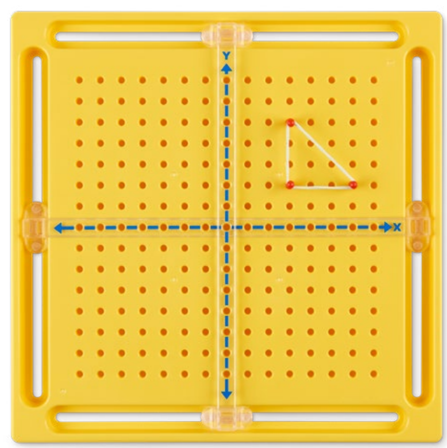


Figure 1

4. Now that a complete rotation has been demonstrated, direct students to write in "counterclockwise" in the first blank of the first sentence on their worksheet. As for the second blank, tell them that since they are rotating quadrant to quadrant, it is important to also realize that they are rotating all of their figures around one specific fixed point. That point is the origin of the plane. Remind them that the origin is (0, 0). Direct them to write in "origin" on the second blank of the first sentence.

Activity cont.

1. Ask students what shape is being made with the rotations if they are moving counterclockwise. If they struggle to come up with the answer of a circle, feel free to show students visually with a finger to help them out. Ask how many degrees are in a circle ***(360°)***. Have students get together with a partner to determine how many degrees were in each of the rotations. Students should be able to determine that a total of four rotations were made. 360 ÷ 4 = 90, so each individual rotation was 90°. From the original figure, there was a 90°, 180°, and 270° rotation. Have students record those observations on their worksheet.
2. Now that some rules have been established, students are ready to begin to actually rotate the figure. See if students can tell you which quadrant they will make the first rotation to ***(Quadrant II)*** and the measurement of the rotation ***(90°)***. Direct students to write down the coordinates for the rotated shape on the line for Quadrant II under Figure 1 on their worksheet as you provide them. The coordinates are (–2, 3), (–2, 6), and (–5, 3). See the answer key for a visual.
3. Students should use blue pegs to plot those points on their geoboard, then draw the figure in Quadrant II of the plane on their worksheet. Have students talk with a partner about what they notice about the relationship between the points in Quadrant I and Quadrant II when they make a 90° rotation. They should also talk about what the rotated shape looks like and its location within the coordinate plane. As students make observations, record them on the board to refer back to in later problems. You may want to create a three-column chart for each type of rotation (90°, 180°, and 270°). Observations students may make include:

   The *x* and *y* coordinates have flipped.
   The new *x*-coordinate is now the negative *y*-coordinate of the original coordinates.
   The *y*-coordinate is now the original *x*-coordinate.
   The rotated figure looks a lot like a reflected figure.
   The triangle is now one row higher than the original triangle in Quadrant I.
4. Give the students the coordinates for the next rotation, and be sure the students are writing them down on the line for Quadrant III under Figure 1 of their worksheet at the same time. The coordinates are (–3, –2), (–6, –2), and (–3, –5). See the answer key for a visual. Before students plot the figure, see if they know which quadrant they will be rotating to ***(Quadrant III)*** and the measurement of this rotation from Quadrant I ***(180°)***. Students should use red pegs to plot those points on their geoboard, then draw the figure in Quadrant III of the plane on their worksheet.
5. Have students talk with their partner about what they notice about the relationship between the points in Quadrant I and Quadrant III when they make a 180° rotation. They should also talk about the appearance of the rotated shape and its location within the coordinate plane. Be sure to record these observations on the board for future reference. Observations students may make include:

   The *x* and *y* coordinates are now the exact opposites of the original coordinates.
   The *x*-coordinate is the negative of the original *x*-coordinate.
   The *y*-coordinate is the negative of the original *x*-coordinate.
   The rotated figure is now an upside-down version of the original figure.
6. Give the students the coordinates for the final rotation. Make sure students write them down on the line for Quadrant IV under Figure 1 of their worksheet as you do so. The coordinates are (2, –3), (2, –6), and (5, –3). See the answer key for a visual. Before students plot the figure, see if they know which quadrant they will be rotating to ***(Quadrant IV)*** and the measurement of this rotation from Quadrant I ***(270°)***. Students should use blue pegs to plot those points on their geoboard, then draw the figure in Quadrant IV of the plane on their worksheet.
7. Have students talk with their partner about what they notice about the relationship between the points in Quadrant I and Quadrant IV when they make a 270° rotation. They should also talk about the appearance of the rotated shape and its location within the coordinate plane. Be sure to record these observations on the board for future reference. Observations students may make include:

   The *x* and *y* coordinates have flipped again.
   The *x*-coordinate is now the original *y*-coordinate.
   The *y*-coordinate is now the negative of the original *x*-coordinate.
   The rotated figure is now what the original figure reflected across the *x*-axis would look like.
8. Tell students that they have come up with a lot of interesting observations regarding the coordinates in the rotated triangle figure, and now they will see if some or all of these observations still apply when they begin in a different quadrant and use a different figure. Students should clear their geoboard, then use red pegs to create this quadrilateral in Quadrant III: (–1, –2), (–1, –5), (–4, –5), and (–3, –3). Students should also write these coordinates under the Figure 2 coordinate plane for Quadrant III and draw the figure in Quadrant III of that same plane.
9. Just as with the triangle, students will make a 90° rotation. Prompt them to tell you the axis over which the rotation will cross ***(y-axis)*** and the quadrant it will be in ***(Quadrant IV)***. Review the observations students made for the 90° rotation for the triangle. Provide the coordinates for the 90° rotation while students record them on their worksheet for Quadrant IV of Figure 2. The rotated coordinates are (2, –1), (5, –1), (5, –4), and (3, –3). See the answer key for a visual. Students should plot the figure on their geoboard and draw the figure on the worksheet.

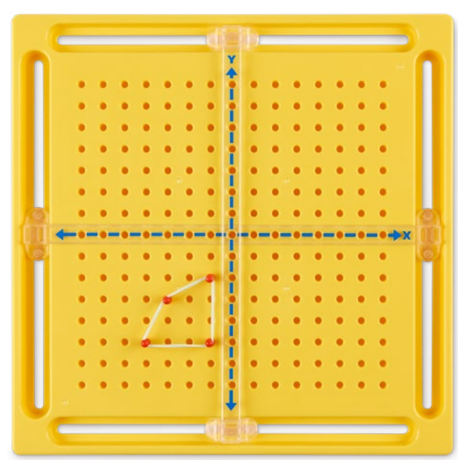


Figure 2

10. Ask students what they notice. They should be able to connect that the same kind of 90° rotation changes occur with this figure. The new *x*-coordinate is the opposite of the original *y*-coordinate. The new *y*-coordinate is the original *x*-coordinate. Ask if there is a rule that could be created for 90° rotations. Students should be able to see that the new coordinates are the ***(–y, x)*** of the original coordinates. Have them record that rule on the student worksheet for 90° rotations.

Activity cont.

1. Ask students which axis will be crossed next as they continue to rotate around the origin ***(the x-axis)*** and what quadrant they will be in ***(Quadrant I)***. They should also be able to tell you that the degree of rotation from the original quadrilateral is 180°. Give students the rotated coordinates for the quadrilateral's 180° rotation, making sure they record them on their worksheet for Quadrant I of Figure 2. The coordinates are (1, 2), (1, 5), (4, 5), and (3, 3). See answer key for a visual. Students should plot the figure on their geoboard and draw the figure on their worksheet.
2. Ask students what they notice. They should be able to connect that the same kind of 180° rotation occurs with this figure, meaning that the new *x*-coordinate is the opposite of the original *x*-coordinate and the new *y*-coordinate is the opposite of the original y-coordinate. See if students can devise a rule for 180° rotations. They should be able to see that the new coordinates are the ***(–x, –y)*** of the original coordinates. Have them record that rule on the worksheet for 180° rotations.
3. Point out that they are seeing some rules that are true for all rotations around the origin, then see if they know which axis will be crossed on the final rotation ***(the y-axis)***, the quadrant they will be in ***(Quadrant II)***, and the degree of rotation from the original quadrilateral ***(270°)***. At this time, if students seem to be catching on, consider having them determine the final set of coordinates on their own or with a partner instead of you listing them. Whether they come up with them on their own or you provide them, they should write the rotated coordinates on their worksheet for Quadrant II of Figure 2. The rotated coordinates are (–2, 1), (–5, 1), (–5, 4), and (–3, 3). See answer key for visual. Students should also plot the figure on their geoboard and draw the figure on their worksheet.
4. Ask students what they notice. They should be able to connect that the same kind of 270° rotation changes occur with this figure. The new *x*-coordinate is the original *y*-coordinate. The new *y*-coordinate is the opposite of the original *x*-coordinate. See if students can devise a rule for 270° rotations. They should be able to see that the new coordinates are ***(y, –x)*** of the original coordinates. Have them record that rule on the worksheet for 270° rotations.

Check for understanding

Tell students that you will give them the coordinates for another triangle. Using the discoveries they've made during their investigations today, they need to determine the coordinates of the shape when it is rotated 90°, 180°, and 270°. As before, they should have coordinates for four total shapes written and four total shapes created on both their geoboards and on the coordinate plane of their worksheet. The original triangle they need to create is in Quadrant I at the coordinates of (5, 1), (6, 7), and (4, 5). Give students time to work independently, then ask the following questions:

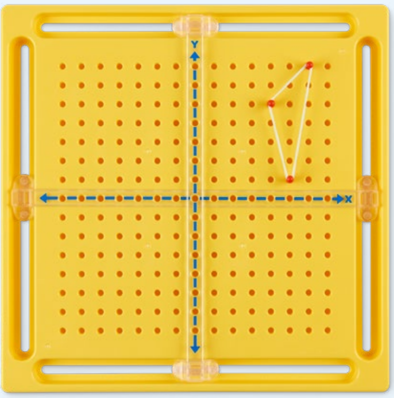


Figure 3

1. When you make a 90° rotation, what is the rule? ***(The new x-coordinate is the opposite of the original y-coordinate. The new y-coordinate is the original x-coordinate.)***
2. What is the rotated coordinate pair for (5, 1)? ***(–1, 5)***
3. What is the rotated coordinate pair for (6, 7)? ***(–7, 6)***
4. What is the rotated coordinate pair for (4, 5)? ***(–5, 4)***
5. What axis have you crossed, what is your new quadrant, and what degree of rotation have you gone? ***(I have crossed the y-axis into Quadrant II with a 90° rotation.)***
6. When you make a 180° rotation, what is the rule? ***(The new x-coordinate is the opposite of the original x-coordinate. The new y-coordinate is the opposite of the original y-coordinate.)***
7. What is the rotated coordinate pair for (5, 1)? ***(–5, –1)***
8. What is the rotated coordinate pair for (6, 7)? ***(–6, –7)***
9. What is the rotated coordinate pair for (4, 5)? ***(–4, –5)***
10. What axis have you crossed, what is your new quadrant, and what degree of rotation have you gone? ***(I have crossed the x-axis into Quadrant III with a 180° rotation.)***
11. When you make a 270° rotation, what is the rule? ***(The new x-coordinate is the opposite of the original x-coordinate. The new y-coordinate is the opposite of the original y-coordinate.)***
12. What is the rotated coordinate pair for (5, 1)? ***(1, –5)***
13. What is the rotated coordinate pair for (6, 7)? ***(7, –6)***
14. What is the rotated coordinate pair for (4, 5)? ***(5, –4)***
15. What axis have you crossed, what is your quadrant, and what degree of rotation have you gone? ***(I have crossed the y-axis into Quadrant IV with a 270° rotation.)***

Students will complete the remainder of the student worksheet independently. Take a moment to explain that Figure 4 is exactly like Figures 1-3. Figures 5-8 only require students to determine one given type of rotation rather than all three rotations. They will use the same skills to determine the coordinates, but they may need an explanation from you for how to complete the first kind of that problem.

Intervention

Help students better see the connection between the type of rotation and the rule of that rotation, start all figures in Quadrant I. By eliminating one element of change, students may be able to better make the connection.

Extension

Students can begin to combine and investigate all three types of transformations. EXAMPLE: Create a triangle in Quadrant I with the coordinates (1, 1), (3, 6), and (5, 3). Reflect that triangle into Quadrant II, then rotate it 180°.

Rotations — **worksheet**

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Name: ____________________ Date: ____________________

Rules of Rotations

When rotating on the coordinate plane, I will always rotate in a ____________________ direction around the __________.

On the coordinate plane, I will rotate ________, ________, or ________.

If I start in Quadrant II...
A ____________ rotation will be in Quadrant _______.
A ____________ rotation will be in Quadrant _______.
A ____________ rotation will be in Quadrant _______.

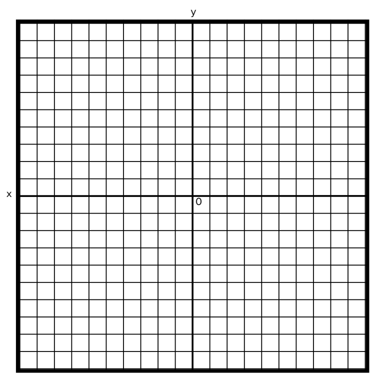
If I start in Quadrant I...
A ____________ rotation will be in Quadrant _______.
A ____________ rotation will be in Quadrant _______.
A____________ rotation will be in Quadrant _______.

If I start in Quadrant III...
A ____________ rotation will be in Quadrant _______.
A ____________ rotation will be in Quadrant _______.
A ____________ rotation will be in Quadrant _______.

If I start in Quadrant IV...
A ____________ rotation will be in Quadrant _______.
A ____________ rotation will be in Quadrant _______.
A ____________ rotation will be in Quadrant _______.

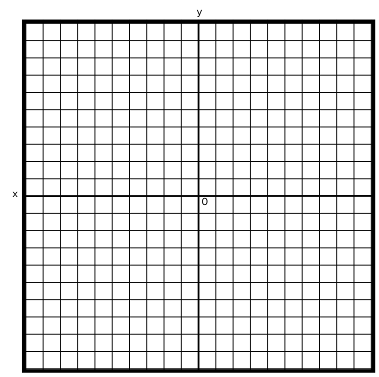
____________ rotation rule: ____
____________ rotation rule: ____
____________ rotation rule: ____

Figure 1



Quadrant I: ______________________________
Quadrant II: ______________________________
Quadrant III: ______________________________
Quadrant IV: ______________________________

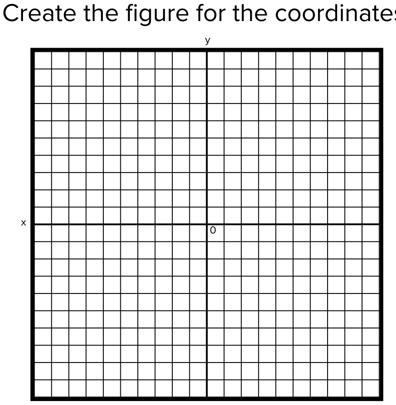
Figure 2



Quadrant I: ______________________________
Quadrant II: ______________________________
Quadrant III: ______________________________
Quadrant IV: ______________________________

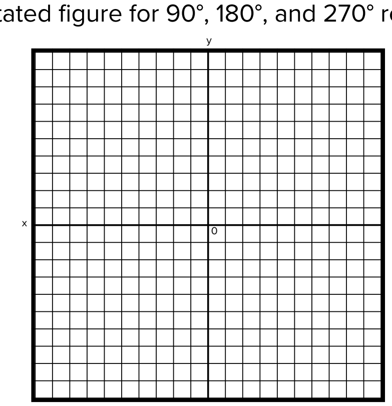
Directions: Create the figure for the coordinates provided, then create the rotated figure for 90°, 180°, and 270° rotations.

Figure 3



Quadrant I: (5, 1), (6, 7), (4, 5)
Quadrant II: ______________________________
Quadrant III: ______________________________
Quadrant IV: ______________________________

Figure 4



Quadrant I: ______________________________
Quadrant II: ______________________________
Quadrant III: ______________________________
Quadrant IV: (4, –1), (3, –3), (5, –3), (4, –6)

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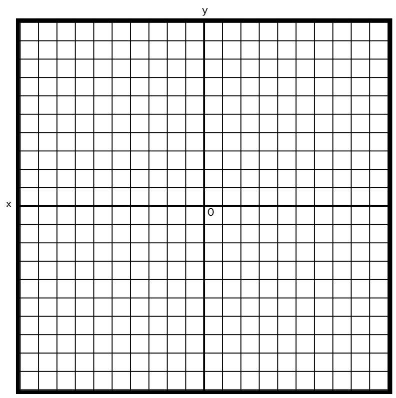
Directions: Create the figure for the coordinates provided, then create the rotated figure for the specific type of rotation listed.

Figure 5

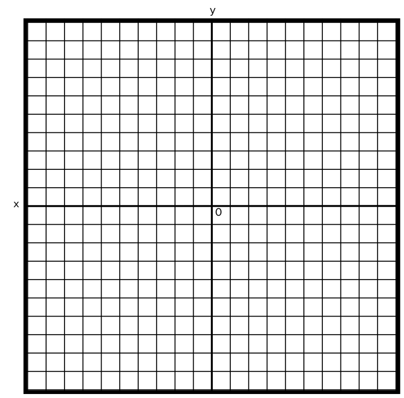


Original Coordinates: (–1, 6), (–4, 5), (–7, 7)

180° Rotation Coordinates:

____

Figure 6

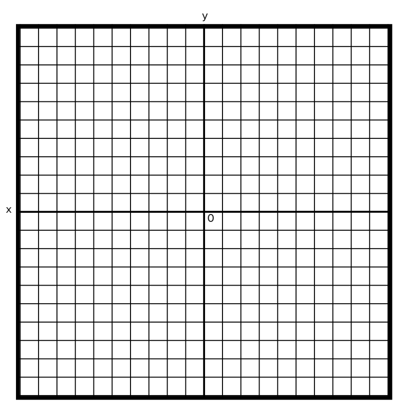


Original Coordinates: (–2, 2), (–4, 2), (–7, 5)

270° Rotation Coordinates:

____

Figure 7

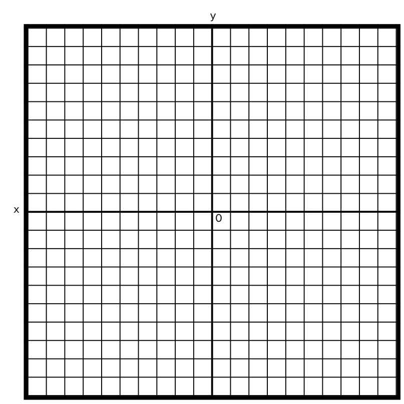


Original Coordinates: (1, –2), (4, –1), (7, –6), (3, –5)

90° Rotation Coordinates:

____

Figure 8



Original Coordinates: (–5, –2), (–6, –5), (–1, –5), (–3, –2)

270° Rotation Coordinates:

____

Rotations — **answer key**

Rules of Rotations

When rotating on the coordinate plane, I will always rotate in a counterclockwise direction around the origin.

On the coordinate plane, I will rotate 90°, 180°, or 270°.

If I start in Quadrant II...

A 90° rotation will be in Quadrant III.

A 180° degree rotation will be in Quadrant IV.

A 270° degree rotation will be in Quadrant I.

If I start in Quadrant I...

A 90° rotation will be in Quadrant II.

A 180° rotation will be in Quadrant III.

A 270° rotation will be in Quadrant IV.

If I start in Quadrant III...

A 90° rotation will be in Quadrant IV.

A 180° rotation will be in Quadrant I.

A 270° rotation will be in Quadrant II.

If I start in Quadrant IV...

A 90° rotation will be in Quadrant I.

A 180° rotation will be in Quadrant II.

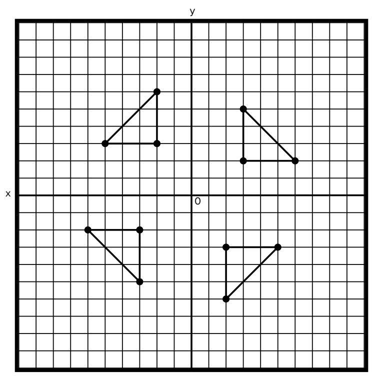
A 270° rotation will be in Quadrant III.

90° rotation rule: The new coordinates are (–y, x) of the original coordinates.

180° rotation rule: The new coordinates are (–x, –y) of the original coordinates.

270° rotation rule: The new coordinates are (y, –x) of the original coordinates.

Figure 1



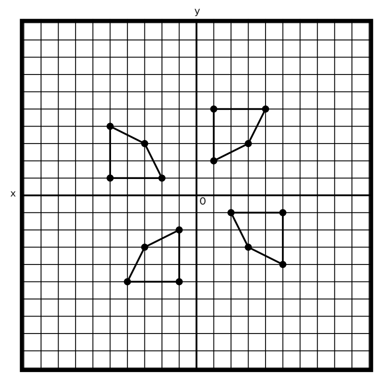
Quadrant I: (3, 2), (6, 2), (3, 5)

Quadrant II: (–2, 3), (–2, 6), (–5, 3)

Quadrant III: (–3, –2), (–6, –2), (–3, –5)

Quadrant IV: (2, –3), (2, –6), (5, –3)

Figure 2



Quadrant I: (1, 2), (1, 5), (4, 5), (3, 3)

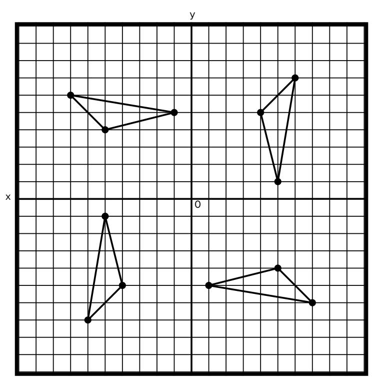
Quadrant II: (–2, 1), (–5, 1), (–5, 4), (–3, 3)

Quadrant III: (–1, –2), (–1, –5), (–4, –5), (–3, –3)

Quadrant IV: (2, –1), (5, –1), (5, –4), (3, –3)

Directions: Create the figure for the coordinates provided, then create the rotated figure for 90°, 180°, and 270° rotations.

Figure 3



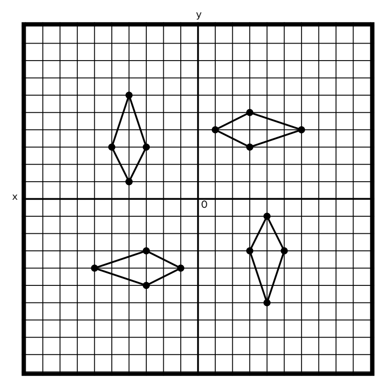
Quadrant I: (5, 1), (6, 7), (4, 5)

Quadrant II: (–1, 5), (–7, 6), (–5, 4)

Quadrant III: (–5, –1), (–6, –7), (–4, –5)

Quadrant IV: (1, –5), (7, –6), (5, –4)

Figure 4



Quadrant I: (1, 4), (3, 3), (3, 5), (6, 4)

Quadrant II: (–4, 1), (–3, 3), (–5, 3), (–4, 6)

Quadrant III: (–1, –4), (–3, –3), (–3, –5), (–6, –4)

Quadrant IV: (4, –1), (3, –3), (5, –3), (4, –6)

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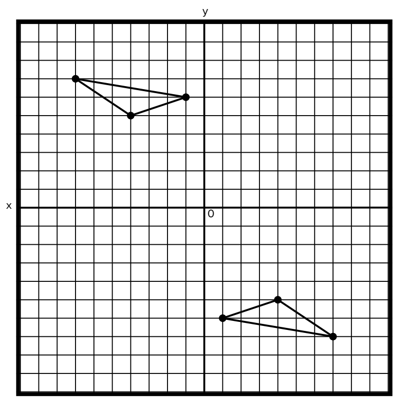
Directions: Create the figure for the coordinates provided, then create the rotated figure for the specific type of rotation listed.

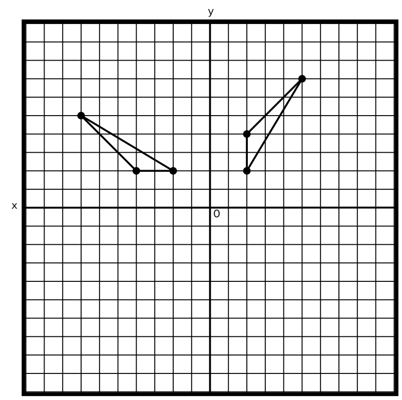
Figure 5



Original Coordinates: (–1, 6), (–4, 5), (–7, 7)

180° Rotation Coordinates:
(1, –6), (4, –5), (7, –7)

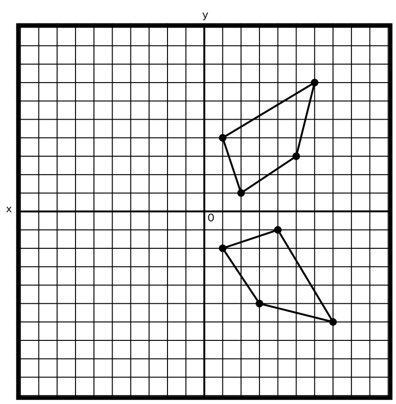
Figure 6



Original Coordinates: (–2, 2), (–4, 2), (–7, 5)

270° Rotation Coordinates:
(2, 2), (2, 4), (5, 7)

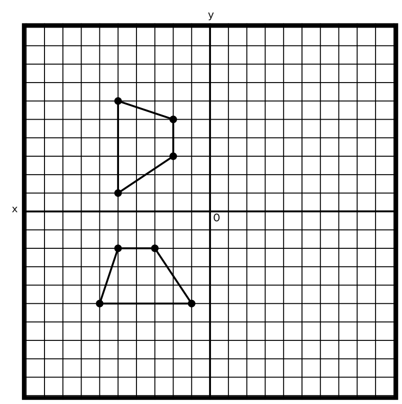
Figure 7



Original Coordinates: (1, –2), (4, –1), (7, –6), (3, –5)

90° Rotation Coordinates:
(2, 1), (1, 4), (6, 7), (5, 3)

Figure 8



Original Coordinates: (–5, –2), (–6, –5), (–1, –5), (–3, –2)

270° Rotation Coordinates:
(–2, 5), (–5, 6), (–5, 1), (–2, 3)